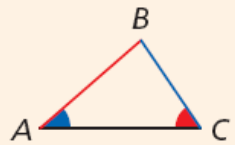
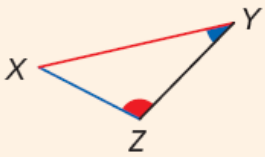
- I can apply inequalities in one triangle.

Vocabulary: Indirect proof

Common Core: CC.9-12.G.CO.10 Prove theorems about triangles.

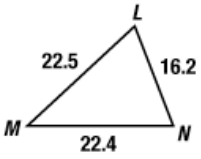
Theorems **Angle-Side Relationships in Triangles**

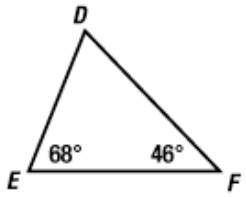
THEOREM	HYPOTHESIS	CONCLUSION
5-5-1 If two sides of a triangle are not congruent, then the larger angle is opposite the longer side. (In △, larger ∠ is opp. longer side.)	 $AB > BC$	$m\angle C > m\angle A$
5-5-2 If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. (In △, longer side is opp. larger ∠.)	 $m\angle Z > m\angle Y$	$XY > XZ$

Refer to video example 2.

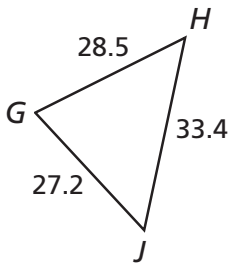
A. Write the angles in order from smallest to largest.

B. Write the side lengths from smallest to largest.





2 Ordering Triangle Side Lengths and Angle Measures



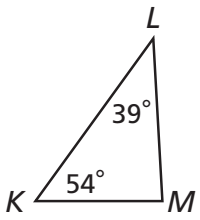
A **Write the angles in order from smallest to largest.**

The shortest side is $\overline{GJ}$, so the smallest angle is $\angle H$.
The longest side is $\overline{HJ}$, so the largest angle is $\angle G$.
The angles from smallest to largest are $\angle H$, $\angle J$, and $\angle G$.

B **Write the sides in order from shortest to longest.**

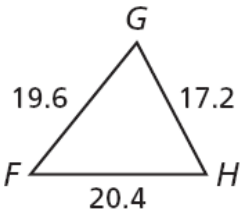
$$m\angle M = 180° - (39° + 54°) = 87° \qquad \triangle \textit{ Sum Thm.}$$

The smallest angle is $\angle L$, so the shortest side is $\overline{KM}$.
The largest angle is $\angle M$, so the longest side is $\overline{KL}$.
The sides from shortest to longest are $\overline{KM}$, $\overline{LM}$, and $\overline{KL}$.

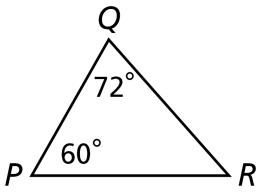


Example 2.

A. Write the angles in order from smallest to largest.

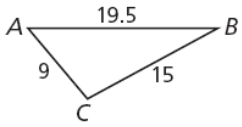


B. Write the sides in order from shortest to longest.

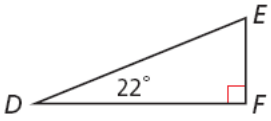


Guided Practice.

5. Write the angles in order from smallest to largest.



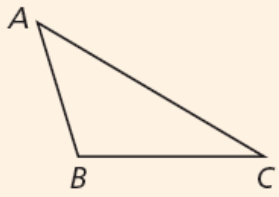
6. Write the sides in order from shortest to longest.



Theorem 5-5-3 **Triangle Inequality Theorem**

The sum of any two side lengths of a triangle is greater than the third side length.

$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



<u>Refer to video example 3</u>. Tell whether a triangle can have the given side lengths.

A. 6, 10, & 14

B. 5, 9, & 14

C. a -1, $a^2 - 2$, and 3a when a = 4

3 Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

A **3, 5, 7**

$3 + 5 \stackrel{?}{>} 7$ $\quad$ $3 + 7 \stackrel{?}{>} 5$ $\quad$ $5 + 7 \stackrel{?}{>} 3$

$8 > 7$ ✓ $\quad$ $10 > 5$ ✓ $\quad$ $12 > 3$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

B **4, 6.5, 11**

$4 + 6.5 \stackrel{?}{>} 11$

$10.5 \not> 11$

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.

C **$n + 5$, n^2, $2n$, when $n = 3$**

Step 1 Evaluate each expression when $n = 3$.

$n + 5$	n^2	$2n$
$3 + 5$	3^2	$2(3)$
8	9	6

Step 2 Compare the lengths.

$8 + 9 \stackrel{?}{>} 6$ $\quad$ $8 + 6 \stackrel{?}{>} 9$ $\quad$ $9 + 6 \stackrel{?}{>} 8$

$17 > 6$ ✓ $\quad$ $14 > 9$ ✓ $\quad$ $15 > 8$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

Example 3. Tell whether a triangle can have sides with the given lengths. Explain.

A. 7, 10, 19 $\quad$ **B.** 2.3, 3.1, 4.6

C. $n + 6$, $n^2 - 1$, $3n$, when $n = 4$.

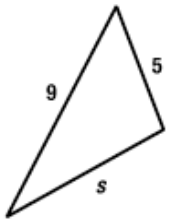
<u>Guided Practice</u>. Tell whether a triangle can have sides with the given lengths. Explain.

7. 8, 13, 21 **8.** 6.2, 7, 9

9. t -2, 4t, $t^2 + 1$, when t = 4.

<u>Refer to video example 4 on page 347.</u>

The lengths of two sides of a triangle are 5 inches and 9 inches. Find the range of possible lengths for the third side.



4 Finding Side Lengths

The lengths of two sides of a triangle are 6 centimeters and 11 centimeters. Find the range of possible lengths for the third side.

Let s represent the length of the third side. Then apply the Triangle Inequality Theorem.

$s + 6 > 11$ $\quad$ $s + 11 > 6$ $\quad$ $6 + 11 > s$

$s > 5$ $\quad$ $s > -5$ $\quad$ $17 > s$

Combine the inequalities. So $5 < s < 17$. The length of the third side is greater than 5 centimeters and less than 17 centimeters.

Example 4. The lengths of two sides of a triangle are 8 inches and 13 inches. Find the range of possible lengths for the third side.

10. Guided Practice. The lengths of two sides of a triangle are 22 inches and 17 inches. Find the range of possible lengths for the third side.

Refer to video example 5.

The distance from Austin to Mason is 108 miles, and the distance from Mason to San Antonio is 111 miles. What is the range of distances from Austin to San Antonio?

5 Travel Application

The map shows the approximate distances from San Antonio to Mason and from San Antonio to Austin. What is the range of distances from Mason to Austin?

Let d be the distance from Mason to Austin.

$d + 111 > 78$	$d + 78 > 111$	$111 + 78 > d$	△ *Inequal. Thm.*
$d > -33$	$d > 33$	$189 > d$	*Subtr. Prop. of Inequal.*
	$33 < d < 189$		*Combine the inequalities.*

The distance from Mason to Austin is greater than 33 miles and less than 189 miles.

Example 5. The figure shows the approximate distances between cities in California. What is the range of distances from San Francisco to Oakland?

11. <u>Guided Practice</u>. The distance from San Marcos to Johnson City is 50 miles, and the distance from Seguin to San Marcos is 22 miles. What is the range of distances from Seguin to Johnson City?

5-5 Indirect proof and Inequalities in one triangle (pp 348) 18, 19, 23, 25, 29, 31, 32, 48, 59, 62.